# Writhe and Twist Helicity
# of Quantum Vortex Systems

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## ABSTRACT

Here we show how kinetic helicity, a fundamental invariant of topological fluid dynamics, can be equally defined for quantum vortices under the Gross-Pitaevskii equation and we discuss how the integral definition of helicity in terms of linking numbers admits interpretation in terms of writhe and twist contributions [1]. We use this decomposition to study the evolution of writhe and twist under direct numerical simulation of the GPE evolution of a Hopf link and the following cascade process produced by a series of reconnections [2]. Since for quantum systems helicity remains always zero, and under anti-parallel reconnection writhe remains conserved across each reconnection event [3], twist must always balance changes in writhe. We show how this naturally leads to a continuous reduction of writhe and twist (of opposite sign) towards small-scale unknotted, unlinked planar rings. Moreover we also show [4] how induction of twist may induce the production of new, seconday defects in the system. This may have useful applications in laboratory production of defect entanglement in condensates.

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[4]  Foresti, M. & Ricca, R.L. (2019) Defect production by twist induction as Aharonov-Bohm effect. *Submitted.*